The Projection Matrix

Lecture 5

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Outline

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World Coordinates

Definition (World Coordinate System)

The world coordinate system is the single coordinate system in which all objects are placed when the scene is rendered.

World Coordinates in 2D

- The default world coordinate system is a "square" with $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$, regardless of the size or shape of the window.
- Typically, this is not the best choice.
- To change the world coordinate system, we need a transformation.
- The function `ortho2D()` will produce the appropriate transformation matrix (called the projection matrix), if we specify the coordinates of the window boundaries: left, right, bottom, top.

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The Projection Matrix

- The projection matrix produced by `ortho2D()` is

$$\mathbf{P} = \begin{pmatrix} \frac{2}{r-\ell} & 0 & 0 & -\frac{r+\ell}{r-\ell} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where $\ell =$ left, $r =$ right, $b =$ bottom, $t =$ top,

The Projection Matrix

- Matrix multiplication $\mathbf{X}' = \mathbf{PX}$ will perform the transformation.

$$\begin{pmatrix} x' \\ y' \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{r-\ell} & 0 & 0 & -\frac{r+\ell}{r-\ell} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix}$$

The Projection Matrix

- The default projection matrix uses $\ell = -1$, $r = 1$, $b = -1$, and $t = 1$, which produces the identity matrix.
- Then the projection matrix is

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{I}.$$

- Every point $\mathbf{X}$ is left unchanged: $\mathbf{PX} = \mathbf{IX} = \mathbf{X}$.

The Projection Matrix

- Suppose our scene is drawn in a rectangle with left $= -4$, right $= 4$, bottom $= -3$ and top $= 3$.
- Then the projection matrix is

$$\mathbf{P} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Map the corners $(-4, -3)$, $(4, -3)$, $(4, 3)$, and $(-4, 3)$.
- Map the point $(2, 1)$.

The Projection Matrix

- Suppose our scene is drawn in a rectangle with left $= 0$, right $= 8$, bottom $= 0$ and top $= 4$.
- Then the projection matrix is

$$\mathbf{P} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & -1 \\ 0 & \frac{1}{2} & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Map the point $(4, 2)$.
- Map the point $(2, 1)$.

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The Vertex Shader

- The multiplication by **P** takes place in the vertex shader (because the vertices are stored in the GPU buffer).
- Therefore, we must pass the projection matrix to the vertex shader.
- The shader will multiply it by the vertex to transform it.

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Uniform Shader Variables

- A uniform shader variable is a shader variable whose value does not change during the processing of the vertices of a primitive, i.e., during a call to `glDrawArrays()`.
- Its value is set by the application program and passed to the shader before calling `glDrawArrays()`.

Uniform Shader Variables

Passing a Shader Variable

```
GLint glGetUniformLocation(program, var_name);
```

- In the application program, we must get a shader location for the uniform variable.
- The `glGetUniformLocation()` will return a location.

Uniform Shader Variables

Passing a Shader Variable

```
void glUniform*(location, value);
void glUniform*(location, count, values);
void glUniformMatrix*(location, count, GL_TRUE, values);
```

- The functions `glUniform*()` and `glUniformMatrix*()` will pass the value(s) to the shaders.
- The third parameter of `glUniformMatrix*()` tells whether the matrix is stored in row-major order (row by row rather than column by column).
- See p. 48 of the Red Book.

Passing the Projection Matrix

Passing the Projection Matrix

```
mat4 proj = ortho2D(left, right, bottom, top);
GLuint proj_loc = glGetUniformLocation(program, "proj");
glUniformMatrix4fv(proj_loc, 1, GL_TRUE, proj);
```

- This code with create the projection matrix and pass it to the shaders.
- `"proj"` is the name of the uniform variable in the shader.
- It is a really good idea to keep the same name in order to avoid confusion.
- Later, we will have many uniform variables.

Using the Projection Matrix

Using the Projection Matrix

```
uniform mat4 proj;

layout (location = 0) in vec2 vPosition;

void main()
{
    gl_Position = proj*vec4(vPosition, 0.0f, 1.0f);
}
```

- In the shader program, we simply declare the variable to be uniform.
- The name must match the name specified in the application program.
- Then multiply it by the position vector and assign to `gl_Position`.

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Assignment

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- Assignment 5, to be turned in by Monday.
- Read pp. 203 - 210, User Transformations.